# Mirror, mirror, on the wall, who in this land is fairest of all? Revisiting the extended concentration index

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**WORK IN PROGRESS!!** 

#### Motivation

How to measure health disparities/inequalities?

#### Common practice:

- o borrow indices from income inequality literature
- Adapt indices to the bivariate setting
- → The concentration index and its extended version
- →often used to evaluate distributional consequences of policies

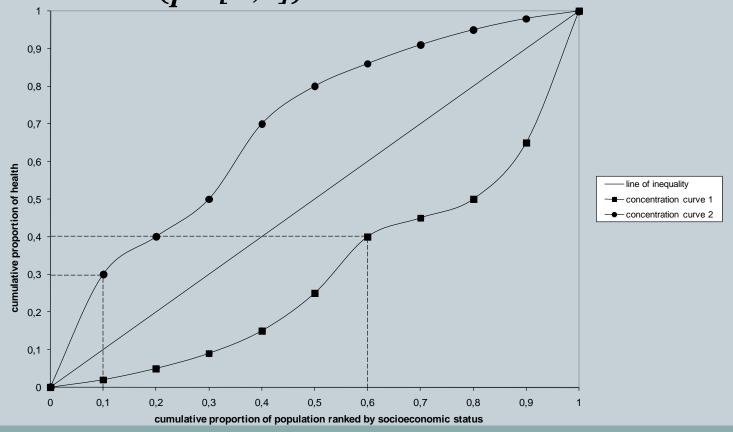
#### But is this sufficient?

- Health is really different → bounded → mirror condition
- What is the meaning of inequality aversion in a bivariate setting?

- Motivation
- Revisiting the concentration index
- Revisiting the extended concentration index
- Revisiting the mirror property
- The generalized extended concentration index
- A symmetry condition
- The symmetric index
- Small-sample bias
- Empirical illustration

### The concentration index revisited (I)

Measuring association between health (h) and income rank  $(p\hat{l} \ [0,1])$ 



### The concentration index revisited (II)

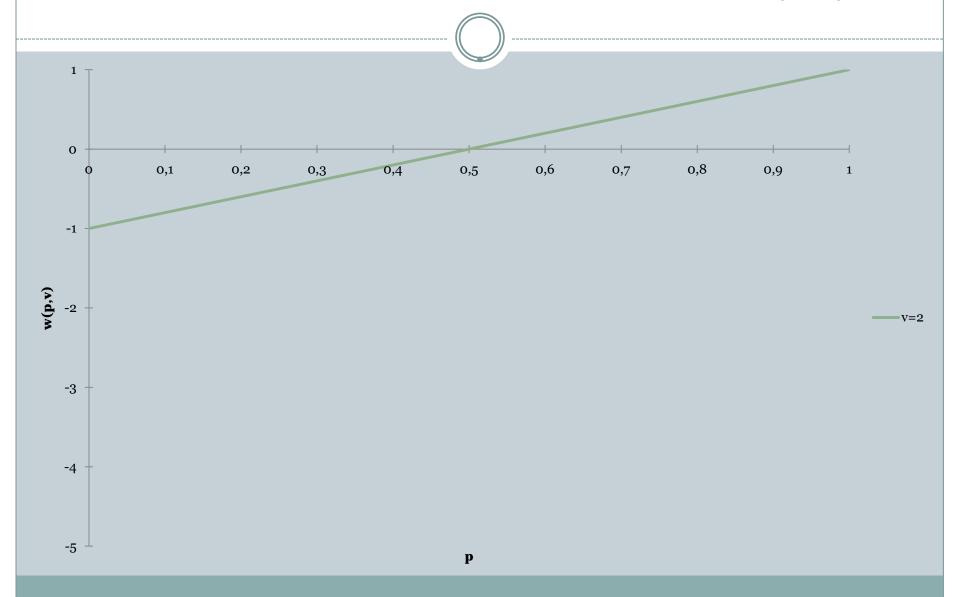
a weighted average of health shares!

$$C(h,p) = \underbrace{\frac{1}{\overline{h}}}_{\substack{normalisation \\ function}} \int_{0}^{1} \underbrace{(2p-1)}_{\substack{weighting \\ function}} \underbrace{h(p)}_{\substack{health \\ levels}} dp$$

• The weighting function increases linearly from 1 to -1 and equals zero for p=0.5

• The concentration index lies between -1 and 1

## The concentration index revisited (III)



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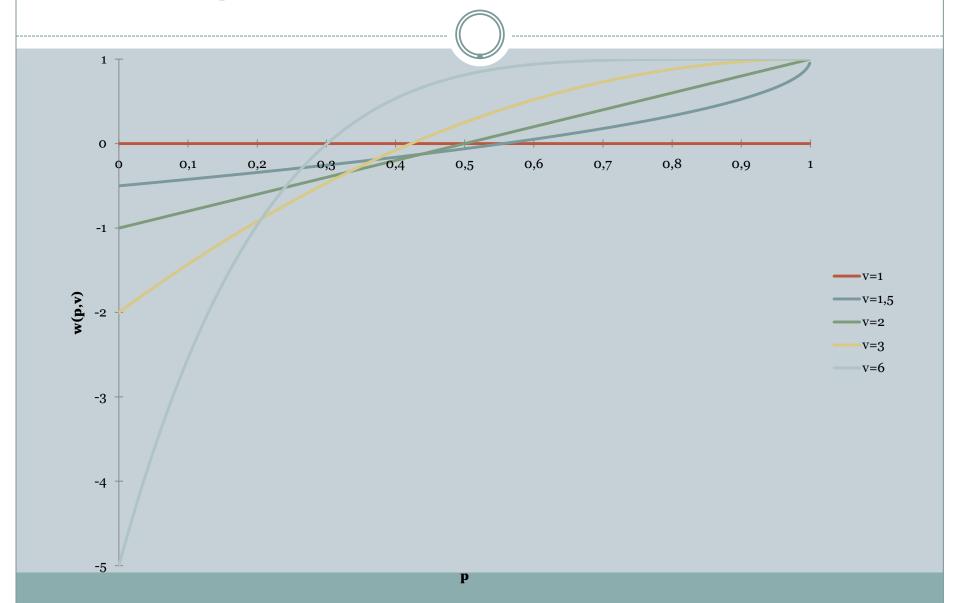
#### The extended concentration index revisited (I)

• Goal: augment the concentration index with a distributional parameter v > 1 reflecting aversion to inequality (e.g. put less/more emphasis on poorest)

$$C(h, p, v) = \frac{1}{\overline{h}} \int_{0}^{1} \left[ 1 - v(1 - p)^{v-1} \right] h(p) dp$$
weighting function

- If v=2, we get the standard concentration index; higher values of v give more negative weight to the poor
- Asymmetric bounds: [1-v, 1]

### Revisiting the extended concentration index (II)

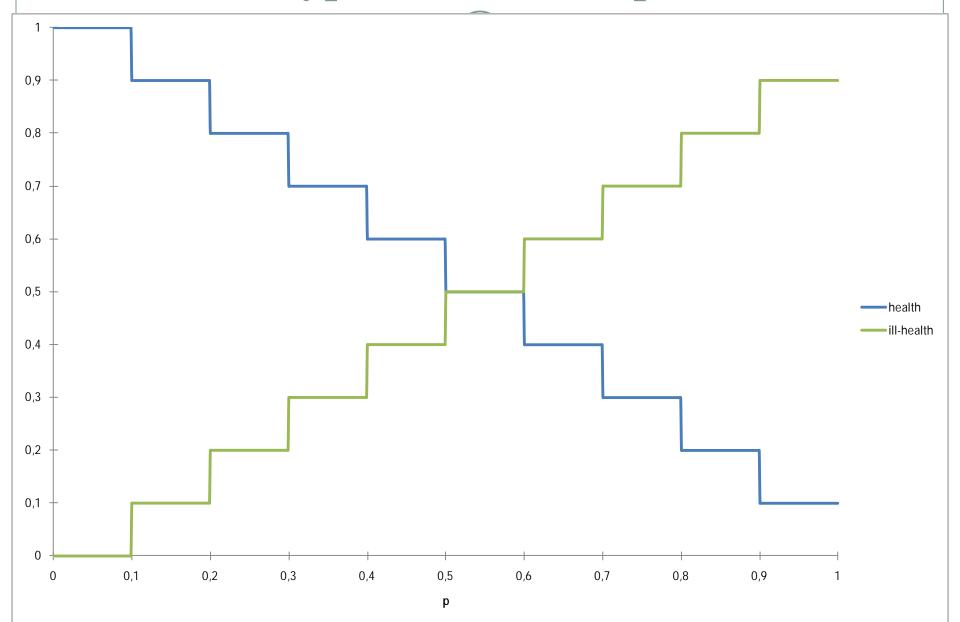


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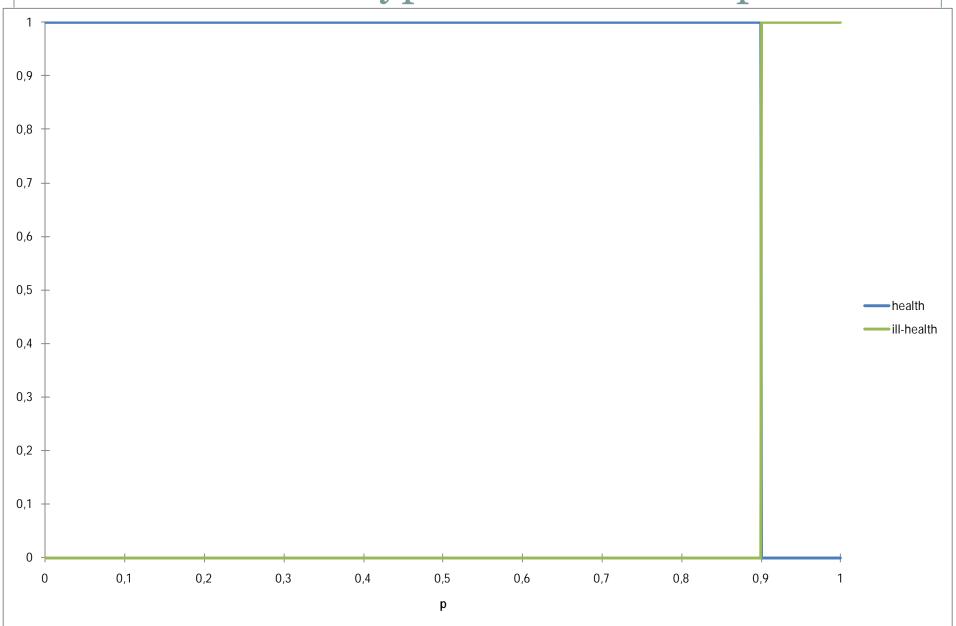
### Revisiting the mirror property (I)

- Health is bounded → two points of view:
  - o Positive side: focus on 'good health' h(p)
  - o Negative side: focus on 'ill health'  $s(p) = h^{max} h(p)$
  - $h(p) \in [0,1]$
- Mirror: health inequality = ill-health inequality
- Violated by the concentration index
  - o Only richest is healthy, versus everyone, except richest, is ill
  - It assumes  $h^{max} = +\infty$
  - o Explains 'stylized facts' in epidemiology

## Hypothetical example



## Extremer hypothethical example



### Revisiting the mirror property (II)

- The violation carries over to the extended index
  - Many applications to both health and ill-health

 First research question: Can we modify the extended concentration index such that it satisfies the mirror property?

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### The generalized concentration index

 Mirror property holds if normalization function is same for health and ill-health

• Solution: make normalization function independent of average health

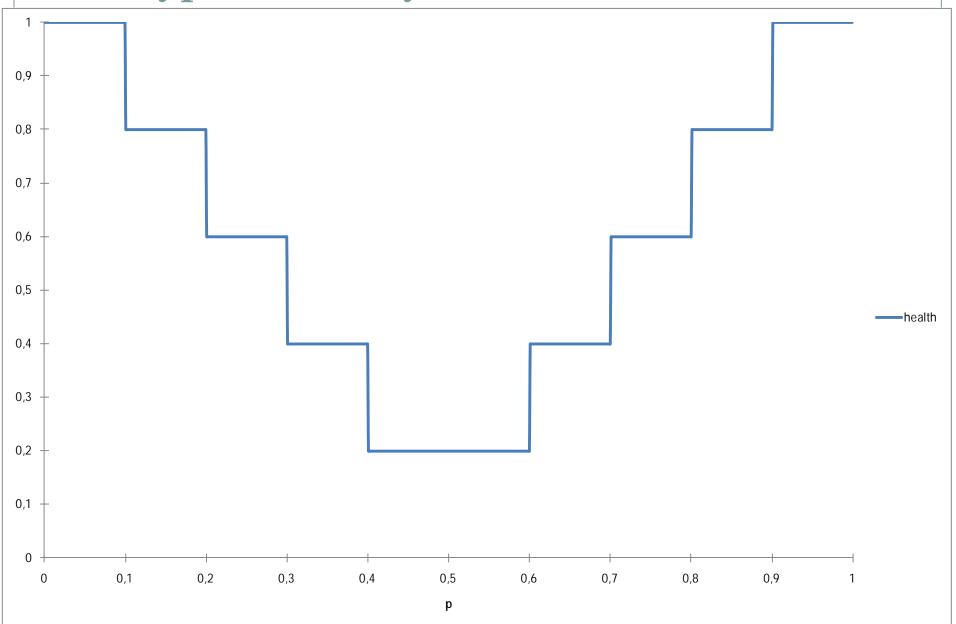
$$GC(h, p, v) = \underbrace{\frac{v^{\frac{v}{v-1}}}{v-1}}_{\substack{normalization \\ function}} \int_{0}^{1} \left[1 - v(1-p)^{v-1}\right] h(p) dp = \underbrace{\frac{v^{\frac{v}{v-1}}}{v-1}}_{\substack{normalization \\ function}} \overline{h}C(h, p, v)$$

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### A symmetry condition

- Chances of having high or low health are symmetrically distributed over the rich and the poor
- 'Symmetric' distribution → no SES health disparities
  - o Only when v=2, otherwise person with weight 0 ≠ the median
  - Intuition: No systematic association between income rank and health!!
- Second research question: can we modify the generalized extended concentration index such that it satisfies the symmetry condition?

## Hypothetical symmetric distribution



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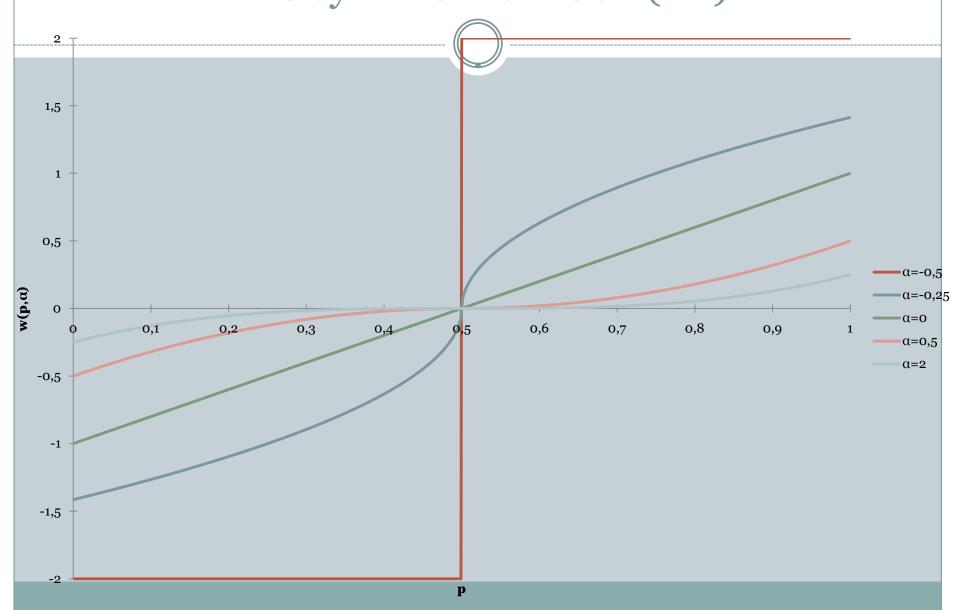
### The symmetric index (I)

- Symmetry condition is satisfied if the weights are symmetric around the median rank 0.5
  - Explains why v=2 is ok
- Solution: normalization function independent of mean health (cf. mirror) and symmetric weighting function

$$S(h, p, \alpha) = \underbrace{(1+\alpha)2^{2(1+\alpha)}}_{normalization} \int_{0}^{1} \underbrace{\left[\left(p-0.5\right)^{2}\right]^{\alpha} \left(2p-1\right)}_{weighting function} h(p) dp$$

• Intuition: Inequality aversion becomes 'extremes aversion' for higher *v*'s





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### Small sample bias

• For relatively small values of n or relatively high values of v and  $\alpha$ , the small-sample bias can be substantial

Bias might be aggravated in case of ties in the income rank

#### • Our solution:

- Very straightforward conceptually
- Reasonably good performance in Monte Carlo simulations

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### Summary of empirical results

#### Demographic Health Surveys for 44 countries

- o Under 5 mortality; and its mirror 5 year survival
- Wealth index constructed using PCA
- Country rankings

#### Summary of findings

- Mirror and symmetry are empirically relevant
- o Small-sample bias and ties are important!

#### Conclusion

- How to incorporate attitudes to inequality into health inequality measurement?
- Prerequisite: mirror
- Symmetry and *not* traditional extensions → aversion to extremes matters in a bivariate setting
- Small sample bias and empirical relevance of methods