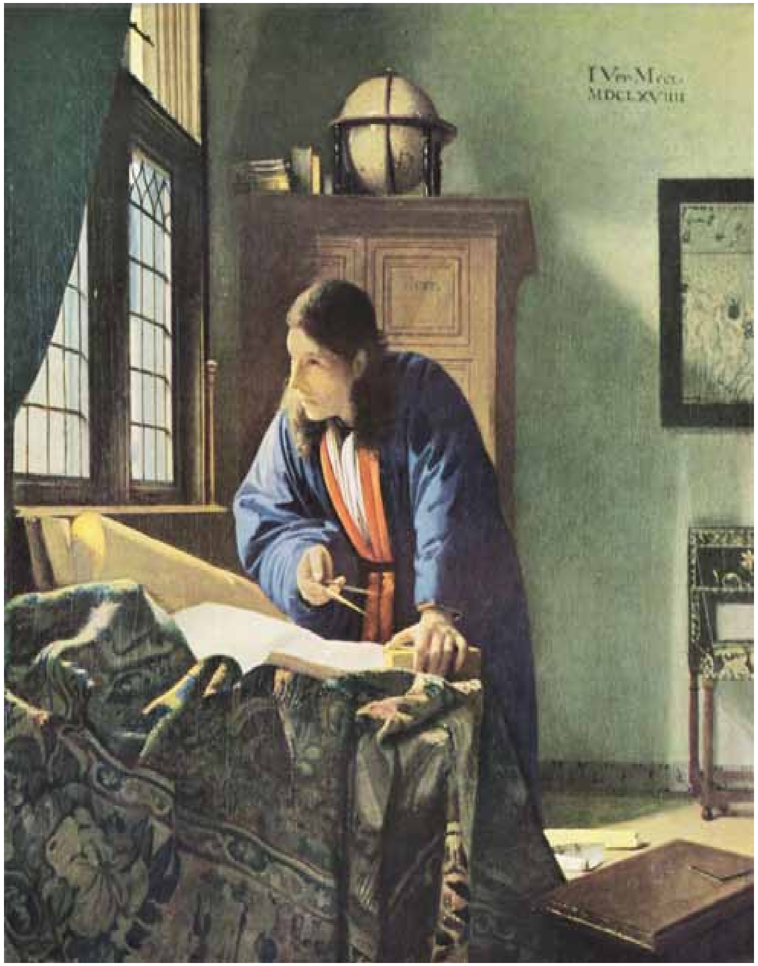
UNIL | Université de Lausanne

2010 ENRGHI
15Th Emerging New Researchers
in the Geography of Health and Impairment Conference

Bidimensional regression beyond goodness-of-fit

Measuring geometric distortions in urban mental maps produced by blind people, wheelchair users and people without disabilities



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The 15th Emerging New Researchers in the Geography of Health and Impairment Conference

10-11 June 2010 -Paris – France

http://www.irdes.fr/Enrghi2010

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Outline

- Bidimensional regression: general presentation
- Conceptual bases
- Goodness-of-fit vs geometric transformations
- Geographical applications: urban mental maps
- Discussion

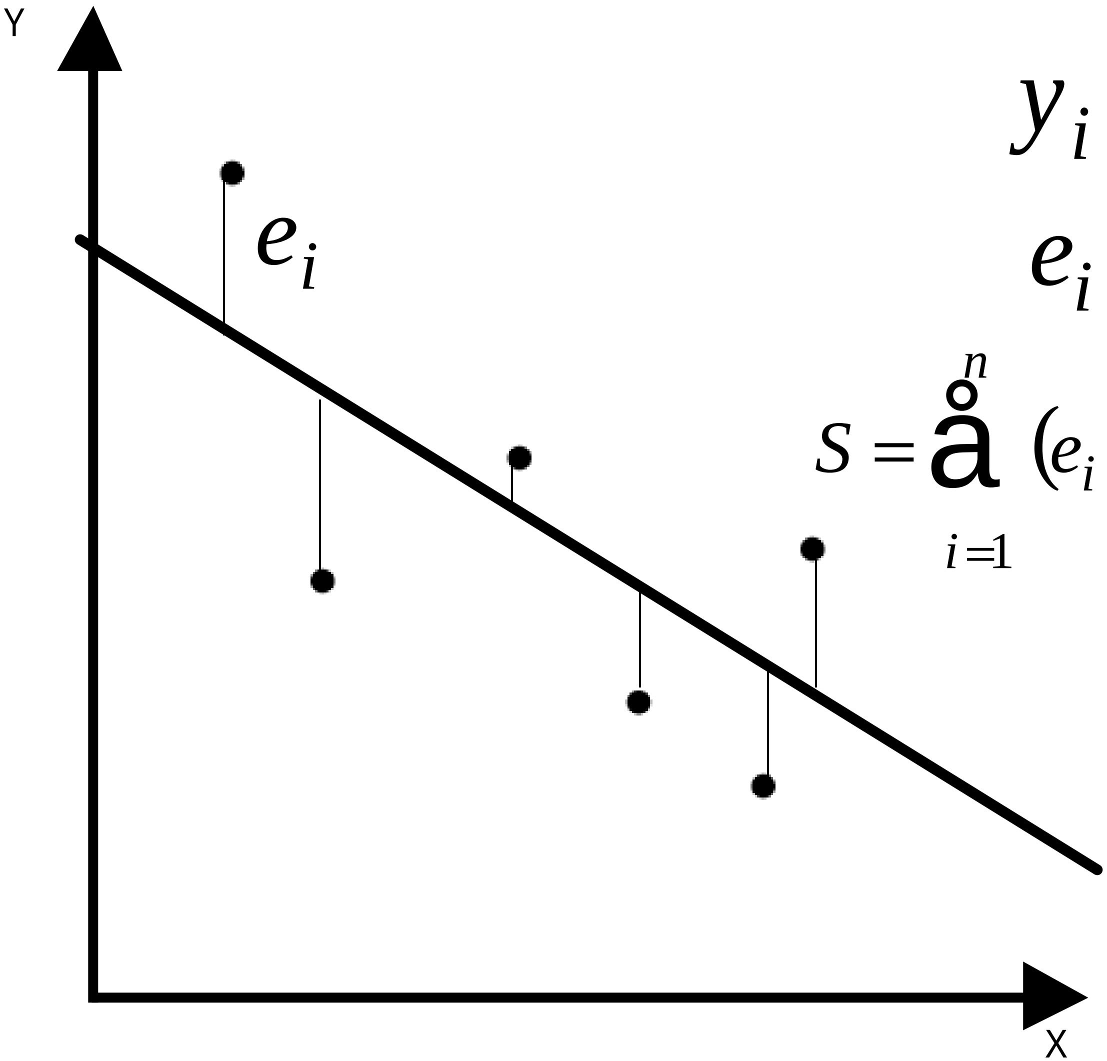
Bidimensional regression

- Assess the similarity between two-dimensional data sets

- Regression analysis and two-dimensional coordinate transformation models

- Tobler introduced it to the geography literature [1965, 1966, 1978, 1994]. Was later introduced to the psychology [Friedman & Kohler, 2003] and computer-science literatures [Kare *et al*., 2008]

Conceptual bases

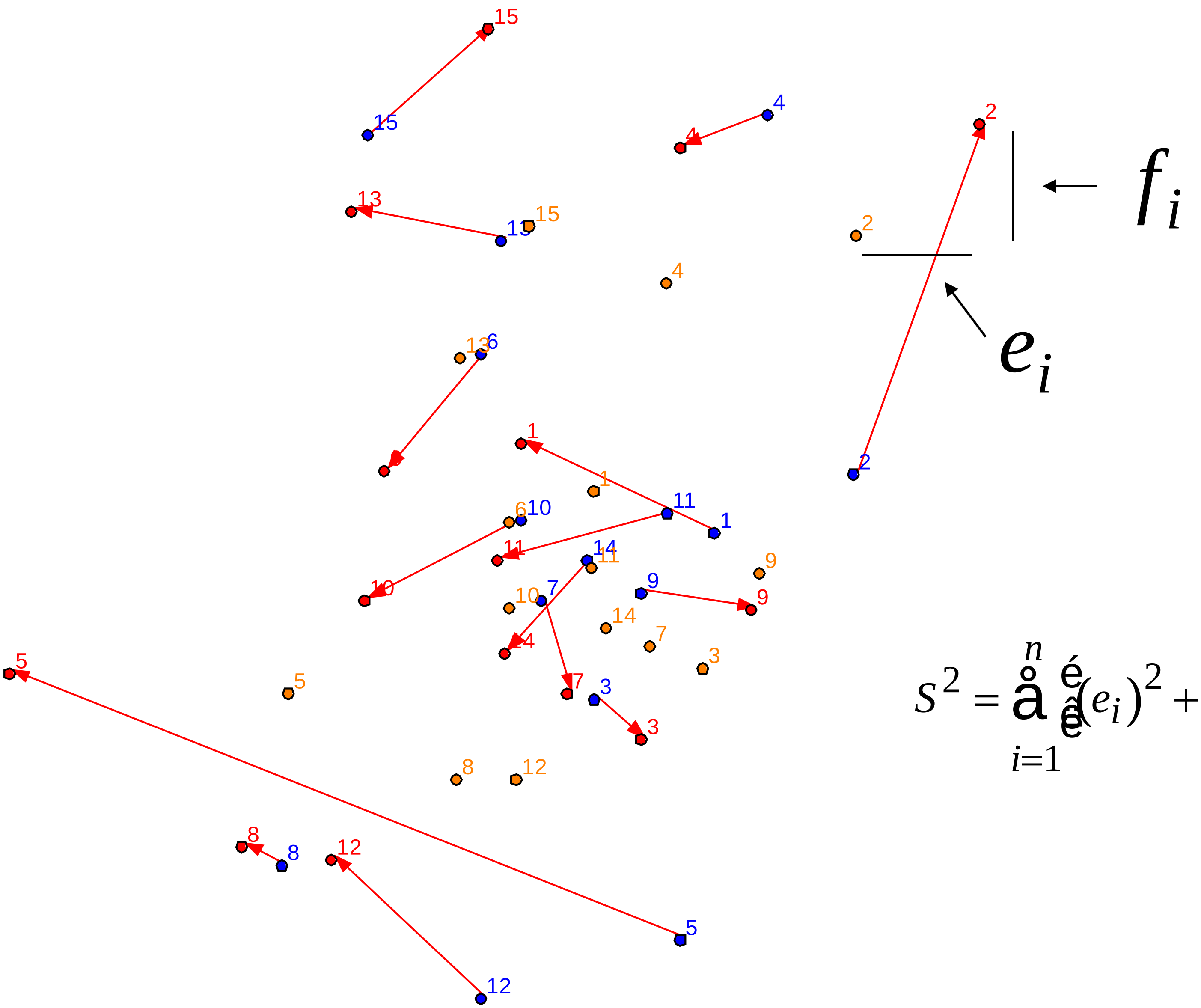


$$y_i = ax_i + b + e_i$$

$$e_i = y_i - ax_i - b$$

$$S = \sum_{i=1}^{n} (e_i)^2 = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

Conceptual bases



$$S^2 = \sum_{i=1}^{n} \left[(e_i)^2 + (f_i)^2 \right]$$

Conceptual bases

General definition of bidimensional regression

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = A \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix} \rightarrow \begin{bmatrix} u_i^* \\ v_i^* \end{bmatrix} = A \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

u_i, v_i = observed coordinates (i.e. dependent coordinates)

A = coordinate transformation matrix

x_i, y_i = reference coordinates (i.e. independent coordinates)

e_i, f_i = residuals

u_i^*, v_i^* = predicted coordinates

Conceptual bases

Euclidean transformation: A=4 parameters

Affine transformation: A=6 parameters

Projective transformation: A=8 parameters

Curvilinear transformation: A=x parameters

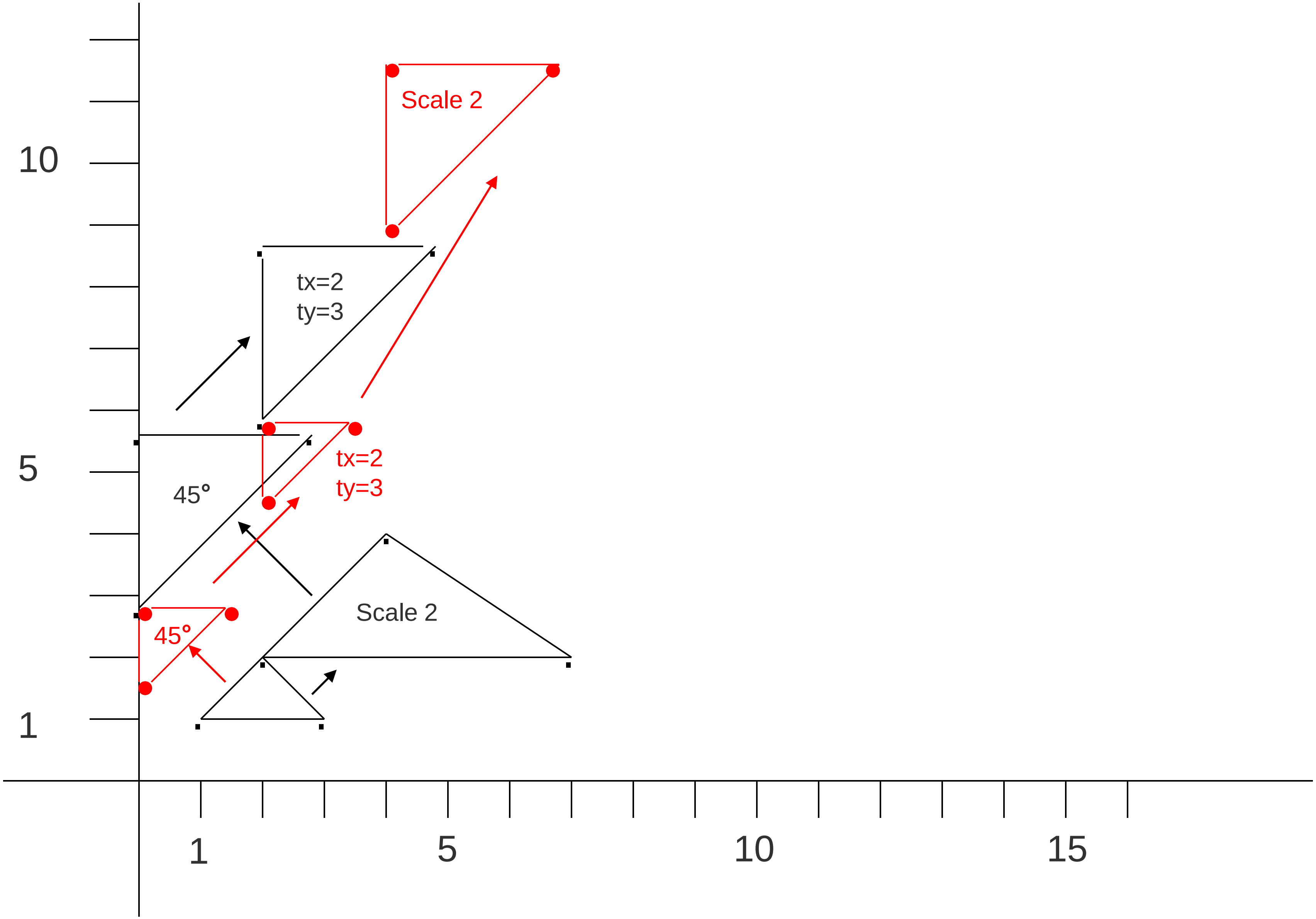
Conceptual bases

Euclidean transformation: A=4 parameters

$$\rightarrow \begin{bmatrix} u^* \\ v^* \end{bmatrix} = \begin{bmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} u^* \\ v^* \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta \\ s\sin\theta & s\cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Geometric transformations



Geometric transformations

1. Scale - rotation - translation = rotation - scale - translation

$$\begin{bmatrix} a_1 & -a_2 & b_1 \\ a_2 & a_1 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} s\cos\theta & -s\sin\theta & tx \\ s\sin\theta & s\cos\theta & ty \\ 0 & 0 & 1 \end{bmatrix}$$

2. Rotation - translation - scale

$$\begin{bmatrix} a_1 & -a_2 & b_1 \\ a_2 & a_1 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} s\cos\theta & -s\sin\theta & stx \\ s\sin\theta & s\cos\theta & sty \\ 0 & 0 & 1 \end{bmatrix}$$

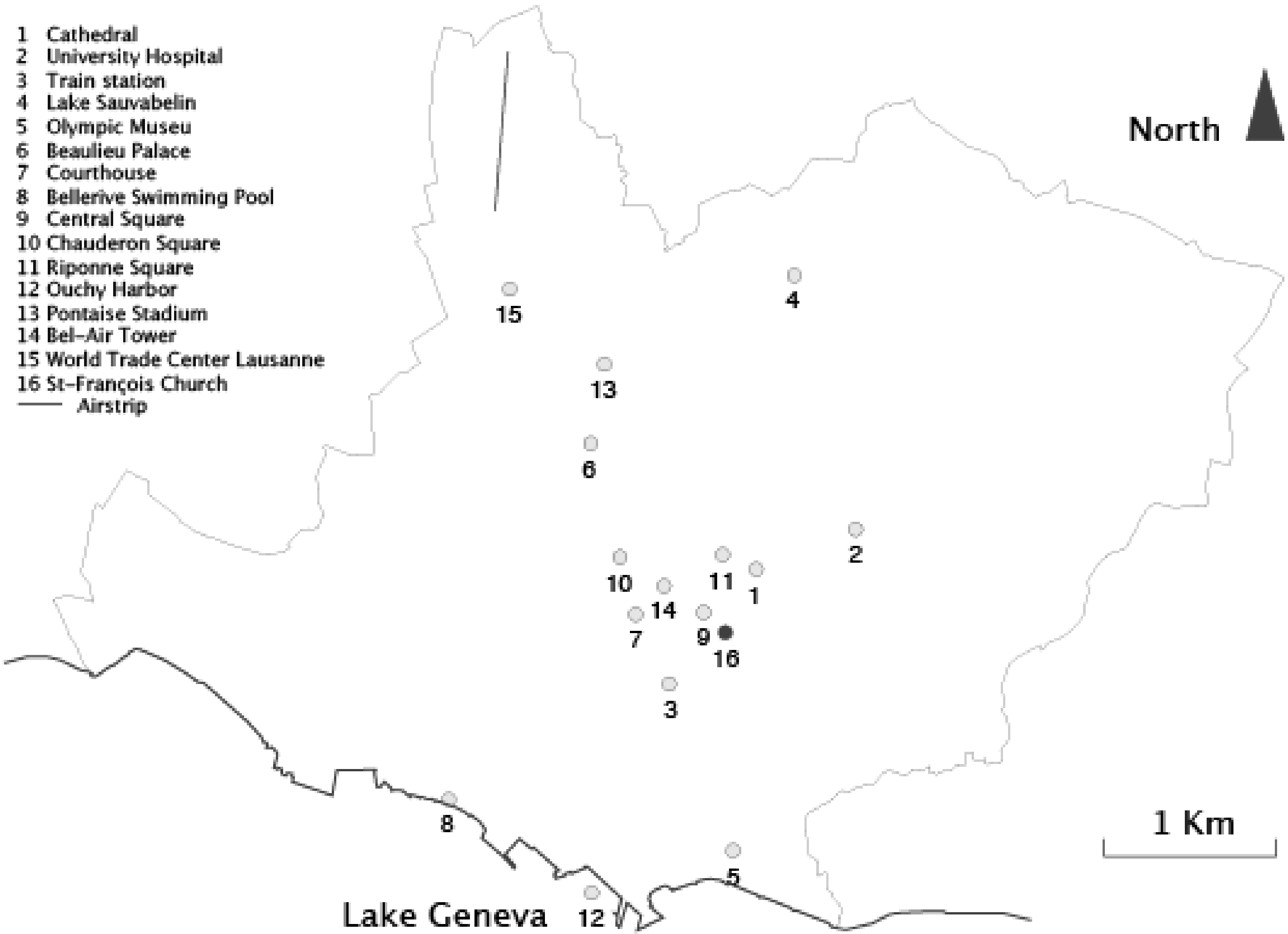
3. Scale - translation - rotation

$$\begin{bmatrix} a_1 & -a_2 & b_1 \\ a_2 & a_1 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} s\cos\theta & -s\sin\theta & tx\cos\theta - ty\sin\theta \\ s\sin\theta & s\cos\theta & tx\sin\theta + ty\cos\theta \\ 0 & 0 & 1 \end{bmatrix}$$

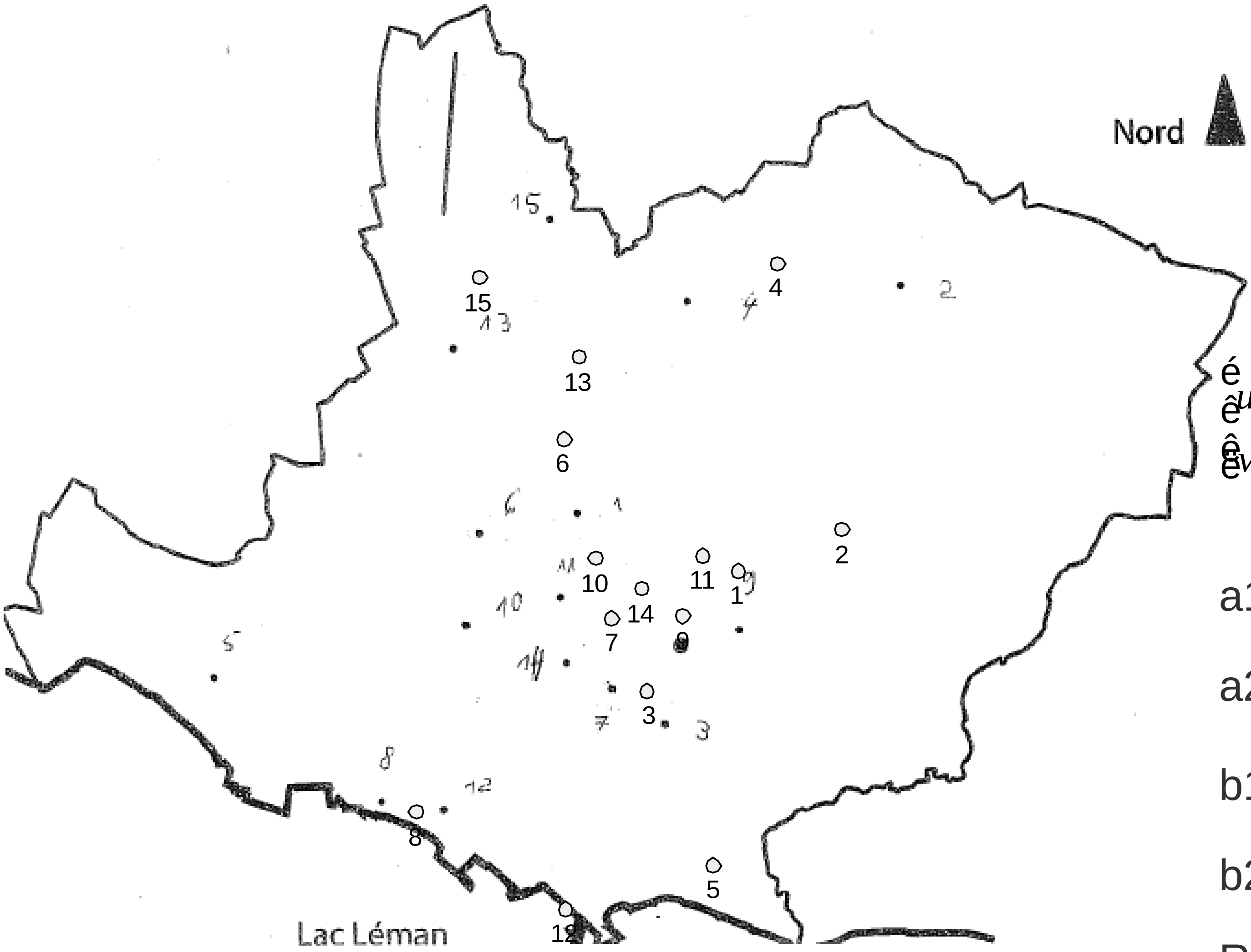
4. Translation - scale - rotation = translation - rotation - scale

$$\begin{bmatrix} a_1 & -a_2 & b_1 \\ a_2 & a_1 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} s\cos\theta & -s\sin\theta & stx\cos\theta - sty\sin\theta \\ s\sin\theta & s\cos\theta & stx\sin\theta + sty\cos\theta \\ 0 & 0 & 1 \end{bmatrix}$$

Urban mental maps



Urban mental maps



$$\begin{bmatrix} u_i^* \\ v_i^* \end{bmatrix} = \begin{bmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

a1=0.86, p=.000

a2=0.14, p=.348

b1=-5.32, p=.837

b2=36.63, p=.164

R2=0.57, p=.000

Urban mental maps

1. Scale - rotation - translation = rotation - scale - translation

scale=0.87, p=0.364; rotation=-9.31°, p=.000; **tx=-5.32**, p=.823; **ty=36.63**, p=.124

2. Rotation - translation - scale

scale=0.87, p=0.364; rotation=-9.31°, p=.000; **tx=-6.09**, p=.817; **ty=41.88**, p=.164

3. Scale - translation - rotation

scale=0.87, p=0.364; rotation=-9.31°, p=.000; **tx=-11.18**, p=.677; **ty=35.28**, p=.115

4. Translation - scale - rotation = translation - rotation - scale

scale=0.87, p=0.364; rotation=-9.31°, p=.000; **tx=-12.78**, p=.662; **ty=40.33**, p=.168

Urban mental maps

$c^2(6) = 9.394, p = .153$

		Transformation order			
		RST(SRT)	RTS	STR	TRS(TSR)
Group	Blind people (n =14)	3	2	3	6
	Wheelchair users (n =14)	5	3	3	3
	Non-impaired (n =14)	3	4	5	2

Discussion

- Algebraic vs geometric parameters

- No a priori order

- “Preferential“ transformation orders

- http://spatial-modelling.info/Darcy-2-module-de-comparaison

Thank you for your attention